For Michael, with best regards Arthur

# Correlations and Physical Locality

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To appear in R. Giere et al (eds) PSA: 1980, Vol II.

## I. Introduction

Correlations between the behavior of pairs of particles are generated in the following experimental situation. An on-line source emits a stream of two-particle systems, where each system is in one and the same quantum state. After emission, the particles - call then (I) and (II) - move off in opposite directions. Each particle then encounters one of several possible barriers that either it passes or doesn't. A short distance behind each barrier is a detector set to register the presence of the particle, should it get that far. Finally, the detectors are connected by a timed relay and counter that registers a "coincidence count" should the two detectors fire within a set, brief time interval. When the experiment is run with various different barriers, detection rates accumulate for each barrier singly, and coincidence rates (the correlations) for the various pairs of barriers. To get at the issues involved it is sufficient to consider two distinct barriers for each particle; say $\tilde{A}$ or $\tilde{A}'$ for particle (I) and $\tilde{B}$ or $\tilde{B}'$ for particle (II). Then from the single and joint rates of detection, one can approximate the single and joint probabilities for passing the barriers. These probabilities are derivable from the fixed quantum state of emission for a particle-pair and I shall assume that these quantum predictions are borne out by the experiments. (The experimental issues here are, in fact, not so straightforward. See [3] for an excellent review of the subtleties involved.)

Thus let

$Q(\tilde{A})$ = the quantum mechanical probability for detecting that particle (I) passes an $\tilde{A}$ - barrier;

$Q(\tilde{B})$ = the quantum mechanical probability for detecting that particle (II) passes a $\tilde{B}$ - barrier; and

$Q(\tilde{A}\tilde{B})$ = the quantum mechanical probability for detecting that in an emitted pair, particle (I) passes an $\tilde{A}$ - barrier and particle (II) passes a $\tilde{B}$ - barrier.

(Similar readings should be given to $Q(\tilde{A}')$, $Q(\tilde{B}')$, $Q(\tilde{A}\tilde{B}')$, $Q(\tilde{A}'\tilde{B}')$, $Q(\tilde{A}'\tilde{B}')$.)

Genuinely puzzling quantum mechanical cases arise where

$$Q(\tilde{A}) = Q(\tilde{B}) = Q(\tilde{A}') = Q(\tilde{B}') = 1/2, \text{ and where}$$

$$Q(\tilde{A}\tilde{B}) = Q(\tilde{A}'\tilde{B}) = Q(\tilde{A}'\tilde{B}') = Q,\ Q(\tilde{A}\tilde{B}') = Q' \text{ for } Q \neq Q';$$

$$0 \leq Q \leq 1/2,\quad 0 \leq Q' \leq 1/2.$$

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Such cases have been produced experimentally, in vindication of the predicted quantum correlations, and they offer the challenge I want to respond to here.

The challenge is this. Can one account for the single and joint probabilities by supposing that they result from a statistical distribution of properties (or dispositions) assigned to the pairs of particles at the source? Such an account is generally called a "hidden variables" theory or model, with the properties (or dispositions) being the hidden variables (or, sometimes, "hidden states"). Since such "hidden" objects are often felt to carry the epistemological burden of unverifiability, I shall try a more neutral-sounding terminology and talk of particle-pairs of various <u>types</u>. Of course if one particle were aware of what barrier is being presented to its partner, it might arrange to alter its behavior so as to produce, in the long run, any correlation whatsoever with its partner. Hence, an integral part of the challenge is to rule out the possi-

bility that in the course of the experiment information as to which barrier one particle encounters (or its causal equivalent) becomes available to the other particle in a way that could affect its own course of behavior. This prohibition is a variety of a no-action-at-a-distance requirement, and could be assimilated to the relativistic requirement that if the separation of the events of the particles passing their respective barriers is space-like then no causal interaction between them should occur. In just this context Einstein referred to the particles as "independent", and to such a requirement as a "principle of separation". (See [12], "Replies . . .", and citations in my [7]) More recently John Bell uses the term "locality" for such a principle, and I shall follow that language, with this proviso. I shall call the intuitive (and vague) no-action-at-a-distance requirement, physical locality and models that respect it physically local. The modifier "physical" is added to distinguish this intuitive, physical condition from the purely mathematical conditions referred to as "locality" in such pronouncements as, "Bell has shown that no local hidden variables theory is consistent with quantum mechanics." The mathematical requirement dubbed "local" here is a kind of factorizability, as we shall see below, and I will so refer to it.

I can now state the main objective of this paper; namely, to examine carefully the requirements of factorizability with a view to showing that factorizability is not necessary for physical locality. That done, I shall go on to suggest some ways of producing models for the correlation experiments that are physically local, although not factorizable, and that can account for all the quantum mechanical single and joint probabilities.

## II. Factorizability: Determinism

In order to introduce the technical concept of factorizability, I must

divide the discussion between the so-called "deterministic" case and the indeterministic one. (It is important, I think, to get the metaphysical picture that goes along with the formalism in each instance.) In both cases we are to picture the pairs of particles emitted by the source as graded into various types $\lambda$, where $\lambda$ ranges over some set $\Lambda$ and is distributed there in accord with a probability density function $\rho$, with $\int_\Lambda \rho(\lambda)d\lambda=1$. The type, $\lambda$, is supposed to provide a complete specification of how each particle in a pair of that type will respond to each of the barriers that it might encounter. (When I speak of the type of a particle, I mean the type of the pair to which it belongs.)

For the deterministic case the type $\lambda$ determines for each particle (in a pair of that type) whether it would pass each barrier. Thus, corresponding to each of the barriers $\tilde{A},\tilde{A}'$, $\tilde{B},\tilde{B}'$, we can introduce a response function $A,A'$, $B$, $B'$ on $\Lambda$ to $\{0,1\}$ where

$A(\lambda) = 1$ iff particle (I) in a pair of type $\lambda$, would pass barrier $\tilde{A}$,

$B(\lambda) = 1$ iff particle (II) in a pair of type $\lambda$, would pass barrier $\tilde{B}$,

and with similar readings for $A'(\lambda)$ and $B'(\lambda)$. To retrieve the quantum single probabilities we require that

$$\text{(RV)} \quad \int_\Lambda S(\lambda)\rho(\lambda)d\lambda = Q(\tilde{S})$$

for $\tilde{S} = \tilde{A},\tilde{A}'$, $\tilde{B},\tilde{B}'$. Thus the response functions are assumed to be $\rho$-measurable. Hence they are random variables on $\Lambda$, with respect to $\rho$, and (RV) simply requires that their distributions duplicate the quantum single probabilities. Put differently, (RV) requires that the quantum single probabilities be obtained by averaging over the $\lambda$-determined responses of each particle.

Factorizability, which the literature glosses as "locality", requires, further, that the joint distributions of the response functions (which, since they

are random variables, will always exist, for all pairs - and multiple distributions for all n- tuples) duplicate, the quantum joint probabilities.

(JD) $\int_\Lambda S(\lambda) \;\; T(\lambda)\rho(\lambda)d\lambda = Q\;(\tilde{S}\tilde{T})$

for (S,T) = (A,B), (A,B'), (A',B) or (A'B'). Put in another way, (JD) requires that the joint probabilities of quantum mechanics (and, presumably, of the correlation experiments) be obtained by averaging over the pairs of $\lambda$ - determined individual responses. For the deterministic case, it is (JD) that constitutes the factorizability requirement.[1]

So, determinism requires that a complete specification $\lambda$ of the emitted pairs settle for each particle whether on not it would pass each barrier. Given that the statistics for a single particle crossing a barrier are obtained by averaging over these pre-arranged responses (RV), factorizability (JD) adds that the correlations are to be obtained by averaging over the joint responses for all emitted pairs.

It is not difficult to see, however, that this probabilistic framework of bivalent random variables, to which determinism plus factorizability (ie, (JD)) commits us, contains inherent limitations on the joint probabilities that can be so represented. For the joint distributions in this framework are constrained by the multiple distributions (such as those for A,A' or for A,B,B', A') which, in the case of quantum mechanics, are not even well defined, since they involve incompatible observables (like spin components in two non-orthogonal directions).

Factorizability makes these multiple distributions relevant here, however, and so we can argue as follows. (Below I write $P(\ )$ for the $\rho$-determined probability that the enclosed random variables take the value 1; where $\bar{A}(\lambda) = 1$ iff $A(\lambda)=0$, etc.)

$$P(\bar{A}A'B) \leqslant P(\bar{A}B) = P(B) - P(AB), \text{ and } P(AA'B) \leqslant P(AA').$$

Adding, and using the fact that $P(\bar{A}A'B) + P(AA'B) = P(A'B)$, yields

$$P(A'B) + P(AB) - P(B) \leqslant P(AA').$$

Similarly,

$$P(\bar{A}A'B') \leqslant P(\bar{A}A'), \text{ and } P(AA'B') \leqslant P(AB').$$

Thus, $\quad P(A'B') - P(AB') \leqslant P(\bar{A}A')$,

Since $\quad P(A') = P(AA') + P(\bar{A}A')$,

(2.1) $\quad P(AB) + P(A'B) + P(A'B') - P(AB') - P(A') - P(B) \leqslant 0.$

Again,

$$P(AA'B') \leqslant P(A'B'), \text{ and } P(AA'\bar{B}') \leqslant P(A\bar{B}') = P(A) - P(AB').$$

So, $\quad P(AA') \leqslant P(A'B') + P(A) - P(AB').$

And,

$$P(\bar{A}A'B) \leqslant P(A'B), \text{ and } P(\bar{A}A'\bar{B}) \leqslant P(\bar{A}\bar{B}) = P(\bar{A}) - P(\bar{A}B) = P(\bar{A}) - P(B) + P(AB).$$

So, $\quad P(\bar{A}A') \leqslant P(A'B) + 1 - P(A) - P(B) + P(AB).$

Hence,

(2.2) $\quad -1 \leqslant P(AB) + P(A'B) + P(A'B') - P(AB') - P(A') - P(B).$

Putting (2.1) and (2.2) together yields

(CH) $\quad -1 \leqslant P(AB) + P(A'B) + P(A'B') - P(AB') - P(A') - P(B) \leqslant 0.$

This is the inequality of [2], derived by Clauser and Horne for the non-deterministic case. Under the special assumptions of the Introduction for the $Q,Q'$ case, this becomes

(SCH) $\quad 0 \leqslant 3Q - Q' \leqslant 1,$

where we use (JD) to identify $P(AB) = Q(\tilde{A}\tilde{B}) = Q$, $P(A'B) = Q(\tilde{A}'\tilde{B}) = Q'$ etc., and (RV) to identify $P(A) = Q(\tilde{A}) = 1/2$ etc.

So, using the constraints placed on the joint distributions by the requirements of marginal probability from the (here) well-defined multiple distributions, the inequality (CH) emerges as a necessary condition that must be satisfied by any joint probabilities for which a deterministic and factorizable theory is possible. (Actually there is a stronger result; namely, that (SCH) is <u>both</u> a <u>necessary</u> <u>and</u> a <u>sufficient</u> condition for a factorizable, deterministic theory, in the Q,Q' case. See my [8].) Thus factorizability, together with the framework of random variables, restricts the class of experiments for which a deterministic model can be given to those whose correlations satisfy (CH). There are quantum mechanical experiments that satisfy these conditions. For example, photon correlation experiments where the barriers are linear polarizers whose relative orientations are the same angle $\Theta$ for A, B and A', B and A', B' and $3\Theta$ for A, B' yield $Q = 1/2 \cos^2 \Theta$ and $Q' = 1/2 \cos^2 3\Theta$. For $\Theta = 39^\circ$ this gives (approximately) $Q = 3/10$ and $Q' = 1/10$, in satisfaction of (SCH). A deterministic model for such an experiment is given by the table below.

| $\lambda$ | $A(\lambda)$ | $A'(\lambda)$ | $B(\lambda)$ | $B'(\lambda)$ |
|---|---|---|---|---|
| $0 \leq \lambda \leq 0.1$ | 1 | 1 | 1 | 1 |
| $0.1 < \lambda \leq 0.3$ | 1 | 1 | 1 | 0 |
| $0.3 < \lambda \leq 0.5$ | 1 | 0 | 0 | 0 |
| $0.5 < \lambda \leq 0.7$ | 0 | 0 | 1 | 1 |
| $0.7 < \lambda \leq 0.9$ | 0 | 1 | 0 | 1 |
| $0.9 < \lambda \leq 1.0$ | 0 | 0 | 0 | 0 |

Here $\lambda$ is assumed to be distributed uniformly ("at random") on [0,1]; i.e., $\rho(\lambda) = 1$ for every $\lambda$. One can readily check that

$$\int_0^1 A(\lambda)d\lambda = \int_0^1 A'(\lambda)d\lambda = \int_0^1 B(\lambda)d\lambda = \int_0^1 B'(\lambda)d\lambda = 1/2,$$

$$\int_0^1 A(\lambda)B(\lambda)d\lambda = \int_0^1 A'(\lambda)B(\lambda)d\lambda = \int_0^1 A'(\lambda)B'(\lambda)d\lambda = 3/10, \text{ and}$$

$$\int_0^1 A(\lambda)B'(\lambda)d\lambda = 1/10.$$

So we can account for the quantum statistics by supposing that the destiny of a particle is fixed at the source by its type, and the performance for that type scheduled by the response functions. Clearly such an account of the experiment requires no exchange of information about the particular barriers encountered.

For $\Theta=60^{\circ}$, however, $Q=1/8$ and $Q'=1/2$, so $3Q-Q'=-1/8$, in violation of (SCH). Thus no factorizable and deterministic model exists for such an experiment. It might be thought, nevertheless, that there could be non-deterministic ones where (CH), as here, fails.

III. Factorizability: Indeterminism

Quantum mechanics makes essential use of probabilities. This suggests that a proper account of the theory leave room for genuinely random events, or the like. That is, that for a quantum system confronted by an experiment having several possible outcomes there is no property (or disposition, or whatever) of the system that determines which particular outcome is realized. Thus for the correlation experiments one might suggest that the best and most complete specification of the properties (etc.) of a particle, as it emerges from the source, could only determine the probabilities for the particle to pass one barrier or another. But whether a given particle does in fact pass a particular barrier should not be fixed in advance even by the items in such a complete description. The passage (or not) is a matter of chance, only the probabilities are fixed.

This suggestion leads to the idea of an indeterminist (or "stochastic") hidden variables theory. Here too we may suppose that the pairs of emitted particles come in various types $\lambda$, also distributed according to a probability density $\rho$, but the idea of indeterminism suggests that the most complete possible specification of the properties (or whatever) of the particles, which is given by the type $\lambda$, only determines the probability for passing each barrier (where, in general, these probabilities are neither 0 nor 1.) Thus we may associate with each of the barriers $\tilde{S}$ a probability function $p(S,\lambda)$ (with $0 \leqslant p(S,\lambda) \leqslant 1$) that gives, for each type $\lambda$, the probability that a particle of that type will pass the indicated barrier. To retrieve the quantum probabilities we require that

$$\text{(IRV)} \qquad \int_\Lambda p(S,\lambda)\,\rho\,(\lambda)d\lambda = Q(\tilde{S})$$

for $\tilde{S} = \tilde{A}, \tilde{B}, \tilde{A}', \tilde{B}'$. To retrieve the quantum correlations we require, in addition, that

$$\text{(IJD)} \qquad \int_\Lambda p(S,\lambda)\cdot p(T,\lambda)\rho\,(\lambda)d\lambda = Q(\tilde{S}\tilde{T}), \text{ for}$$

$$(\tilde{S},\tilde{T}) = (\tilde{A},\tilde{B}),\ (\tilde{A},\tilde{B}'),\ (\tilde{A}',\tilde{B}) \text{ or } (\tilde{A}',\tilde{B}').$$

For the indeterministic case it is (IJD) that constitutes what I am calling the requirement of factorizability. Notice that if all the $p(S,\lambda)$, $p(T,\lambda)$ were either 0 or 1, we should have simple response functions, as in the deterministic case, and then (IRV) and (IJD) would reduce, respectively, to (RV) and (JD). Certainly, then, the concept of a factorizable, indeterministic theory does generalize the concept of a factorizable, deterministic theory. But even more is true. Clauser and Horne show, by a direct algebraic argument, that inequality (CH) is a necessary condition for the existence of a factorizable, indeterministic theory. (There is a pretty way to get the same result; namely, one can show that associated with every indeterministic theory (I), there is a deterministic theory (D) such that (D) is factorizable iff (I) is too. Then since (CH) is a necessary condition for a factorizable (D), it must also be a necessary condition for a factorizable (I). See my [8].) And I have shown in the special Q, Q' case that (SCH) is also a sufficient condition for a factorizable, indeterministic theory. Just to have a concrete example in mind, for the $\Theta=39^\circ$, $Q=3/10$, $Q'=1/10$ case we can produce such an

indeterministic model as follows:

| $\lambda$ | $p(A,\lambda)$ | $p(A',\lambda)$ | $p(B,\lambda)$ | $p(B',\lambda)$ |
|---|---|---|---|---|
| $0\leqslant\lambda\leqslant 1/6$ | 0.887 | 0.887 | 0.887 | 0.113 |
| $1/6<\lambda\leqslant 2/6$ | 0.887 | 0.113 | 0.887 | 0.113 |
| $2/6<\lambda\leqslant 3/6$ | 0.887 | 0.113 | 0.113 | 0.113 |
| $3/6<\lambda\leqslant 4/6$ | 0.113 | 0.887 | 0.887 | 0.887 |
| $4/6<\lambda\leqslant 5/6$ | 0.113 | 0.887 | 0.113 | 0.887 |
| $5/6<\lambda\leqslant 1$ | 0.113 | 0.113 | 0.113 | 0.887 |

Then if $\lambda$ is uniformly distributed on [0,1] we have

$$\int_0^1 p(A,\lambda)d\lambda = 1/2(0.887) + 1/2(0.113) = 1/2 = \int_0^1 p(A',\lambda)d\lambda = \int_0^1 p(B',\lambda)d\lambda = \int_0^1 p(B,\lambda)d\lambda,$$

in satisfaction of (IRV). Also,

$$\int_0^1 p(A,\lambda)\cdot p(B,\lambda)d\lambda = 1/3(0.887)^2+1/3(0.887)(0.113)+1/3(0.113)^2=0.300$$

$$= \int_0^1 p(A',\lambda)p(B,\lambda)d\lambda = \int_0^1 p(A',\lambda)p(B',\lambda)d\lambda; \text{ and}$$

$$\int_0^1 p(A,\lambda)\cdot p(B',\lambda)d\lambda = (0.113)(0.887) = 0.100;$$

in satisfaction of (IJD).

The hope is dashed, however, that the move from determinism to indeterminism will enable one to handle cases like $\Theta=60^\circ$, $Q=1/8$, $Q'=1/2$, provided factorizability is required. But why require it?

For the deterministic case the events of passing or not passing a barrier are represented as random variables on a common space, and factorizability simply requires that the joint distributions of these random variables (which need not be independent variables) yield the quantum mechanical joints. And this may seem a simple and obvious requirement, although I shall challenge it in awhile. In the stochastic case, however, even more is required. For there the rule (IJD) amounts to requiring that for every type $\lambda$, the probability $p(ST,\lambda)$ that particle (I) passes an $\tilde{S}$-barrier and particle (II) passes a $\tilde{T}$-barrier is the product $p(S,\lambda)\cdot p(T,\lambda)$ of the individual probabilities for passage. Thus factorizability here amounts to the requirement of stochastic independence, for every $\lambda$. Hence it forbids that the

events of passage (or not) could be represented as other than stochastically independent events. Since the joint probabilities of quantum mechanics rarely factor in the manner of stochastic independence, the requirement that they be retrieved by averaging over probabilities that do so factor may seem inappropriately strong. Thus the failure of factorizable, indeterminist theories to duplicate the quantum correlations might seem hardly surprising - nor, even, of much interest.

In so describing the results, however, I may fairly be accused of having lost sight of the requirement of physical locality. For if only the probabilities for passing their respective barriers are pre-determined for a pair of particles, and not the events of passage themselves, how could the results of joint passage accumulate except stochastically independently, provided there is no exchange of information that alters the otherwise random events of passage? Thus, in the indeterministic case, the idea seems to be that physical locality requires stochastic independence at every $\lambda$. Despite much effort, however, I have been unable to find any reasonably clear and valid argument that leads from physical locality to factorizability. The most persuasive line of thought comes from recognizing that stochastic independence amounts to requiring that the conditional probability for, say, particle (I) passing its barrier, given that particle II has passed its own, be identical to the original $\lambda$ - determined probability for particle (I) to pass its barrier. Thus, one requires that this $\lambda$ - determined probability not be altered by the events which concern only the other particle. Surely, one could ask, wouldn't such an alteration violate physical locality? But the language of conditional probability here is not really useful; for in speaking of one event happening "given that" another does, it does not distinguish between causal dependencies and mere correlations. So, to reply to the question, why should it not happen that among those pairs that pass certain barriers the relative frequency, say, with which particle (I) turns up is different from its frequency among those pairs where particle (I) passes and particle (II) fails to pass the same barriers? Surely nothing makes this impossible although, to be sure, physical locality rules out a particular sort of causal mechanism that could produce it.

In the deterministic case we understand well enough how correlations could be established at the source, that later produce the statistics of non-independent events. It works, in the manner of other conservation laws, over large distances and without the exchange of causal signals. But we do not seem to understand how similar correlations could be maintained in the indeterministic case where the events themselves are a matter of chance. And so we are driven, really in despair - I think, to ask how, except via causal signals, could such correlations arise. What puzzles us is how there could be a regular pattern to certain pairs of distant events when the occurrence (or not) of each, in every particular case, is not determined. It is as though seeing such a pattern among undetermined events, we feel driven to see it as the result of a causal process.

But this seeming puzzle about correlated joint probabilities is really in line with the central feature of indeterminism itself, which (for the sake of the discussion) I will suppose we accept. For how, one might ask, could there be a regular pattern to passing a barrier (or not) if in every particular case the passage is not determined by anything whatsoever? At the risk of stretching my fragile intuitions - one might say - I could imagine only probability ½ for a chance event. But surely nothing different (e.g. the 0.887 or 0.113 of the preceding model) unless there were an invisible (causal) hand at work. Similarly, for a pair of undetermined events only joint probability ¼ would seem possible, according to even stretched intuitions. So, my line of thought is this. Those who countenance indeterminism in a probabilistic framework, already countenance the emerging of definite, non-random patterns in sequences of undetermined events, and without requiring causal agents to control the generation of the pattern. Why should they then balk at admitting similar non-caused patterns in sequences of pairs of undetermined events? Thus I do not think that the indeterminist can legitimately require stochastic independence, pleading that causal interaction is the only alternative. (Although, I am certainly sympathetic to the temptations that pull him in this direction, for they are the temptations of determinism itself.)

Let me summarize this discussion of indeterminist models. First, there is no good, logical argument to show that factorizability is a consequence of physical locality. Second, the intuitive line of thought that seems to require factorizability amounts to the plea of "What else but a causal hand could establish non-independent correlations?" Third, this intuitive plea runs at cross purposes with indeterminism's basic precept, that no causal hand is required to make order out of apparent chaos - it just happens that way. Thus, neither sound arguments nor sensible intuitions require stochastic independence for joint experiments whose outcomes are neither determined nor causally connected.

I believe this conclusion is correct, despite the widespread statistical practice of assuming stochastic independence in the absence of causal connections. Probability may be a guide to life, but statistical practice - I'm afaid - is no guide to truth. Nevertheless, I do not believe that anything goes; that is, once we see that factorizability is not warranted by physical locality it does not follow that we may assign any old joint probabilities, at $\lambda$, that will average out to the quantum joints (eg., the quantum joints themselves).

## IV. Responsible Indeterminism

The indeterminist cannot be held responsible for providing causal mechanisms to account for the pattern of experimental results, not even for separated joint experiments. However, even the indeterminist must be responsible for respecting the constraints of probability theory. He cannot assign as "probabilities" numbers outside the range of 0 to 1, nor as joint probabilities numbers that exceed the single ones, etc. In the case of the quantum correlations, it is not a matter of assigning joint probabilities to just one, repeatable joint experiment - for which any joint probabilities could be admitted. But, rather, one must assign four joint probability functions to four inter-related joint experiments. The joints assigned to one pair

of experiments must be consistent with those assigned to the other, related pairs, and with the probabilities for the outcomes of any single experiment. The only general way I can think of that will insure such consistency is to require that the single and joint probabilities, for each $\lambda$, be realizable as the single and joint distributions of four random variables (representing the events of passage, or not) on some space or other. Where factorizability is assumed this requirement is automatically satisfied (see my [8] for the proof); so factorizable, indeterminist theories are responsible ones. From the discussion of determinism in Section II, it follows that responsible indeterminism entails inequality (CH) at every $\lambda$. Hence, when we integrate to get the ensemble probabilities, (CH) must hold. Thus (CH) is a necessary condition, and in the Q, Q' case of interest this reduces to the (SCH) inequality. Thus I believe that the restrictions imposed by the (CH) inequality cannot be avoided within the indeterministic framework, even when we relax the requirement of factorizability. So, for $Q=3/10, Q'=1/10$, I think that $p(A,\lambda)=p(B,\lambda)=p(A',\lambda)=p(B',\lambda)=1/2$ and $p(AB,\lambda) = p(A'B,\lambda) = p(A'B',\lambda) = 3/10$, where $p(AB',\lambda) = 1/10$, is a perfectly good indeterminist theory. If we had $Q=4/10$, $Q' = 1/10$, however, and tried the corresponding trick, the result would be irresponsible. For no possible quadruple of bivalent random variables (or "events") could produce such single and joint statistics.

I realize that my line of argument for the requirement of responsible determinism is sketchy (at best), and that it may even appear that I am relying on determinist criteria here, as illegitimate as those I have identified in the background to factorizability. So, let me try to get out these canons of responsibility in quite a different way. (Perhaps this is a "variety of evidence" consideration.)

Let us try to imagine how, when stochastic independence is given up, the requisite correlations might arise out of undetermined events. I don't mean to try

telling a causal story, but I'd like an insightful picture of what might be happening that does not violate physical locality. Think first of a single particle approaching a barrier $\tilde{S}$, with a chance of passing or of not passing, and constrained only by the $\lambda$ - assigned probability $p(S,\lambda)$. I suggest that we picture the outcome of that encounter as the physical realization of the following abstract procedure. The particle carries with it a catalogue of response functions, (say, each defined on [0,1]) one for each barrier it might encounter. These are fixed at the source by $\lambda$ and constitute a set of probabilistic dispositions ("propensities"). The event of passing, say, barrier $\tilde{S}$ then realizes the following procedure: a random selection x is made (say $0 \leqslant x \leqslant 1$) and, from the accompanying catalogue, the response function S is calculated at x, and $S(x)=1$. The event of not passing $\tilde{S}$ realizes a random selection of a number $0 \leqslant x' \leqslant 1$, where $S(x')=0$. I want to emphasize that this picture identifies the passing of a barrier (or not) with a random selection-plus-calculation. Passing barrier $\tilde{S}$ is randomly selecting a "passing" number. Thus, despite the use of response functions and the other apparatus of deterministic models, no determinism is involved in the picture. Nothing determines whether or not the particle will pass any barrier; such passage is a random event. What are fixed in advance are the probabilities for passage, for these correspond to the "size" of the set of passing numbers coded in the $\lambda$ - assigned catalogue. Thus the probability that a particle will pass barrier $\tilde{S}$ is just the probability for randomly selecting a number x from 0 to 1 for which $S(x) = 1$; i.e., the probability is given by $\int_0^1 S(x)dx$.

So this picture of the events, as realizing random selection-plus-calculations, exactly fits the requirements of an indeterminist model: no event is determined in advance, but the probabilities are fixed at the source. What, then, happens in a joint experiment where, say, particle (I) encounters barrier $\tilde{S}$ and particle (II) encounters barrier $\tilde{T}$? The behavior of particle (I) is the random selection of $0 \leqslant x \leqslant 1$ and the calculation of $S(x)$; that of particle (II) is the random delection of $0 \leqslant y \leqslant 1$ and the calculation $T(y)$. Joint passage amounts to $S(x) = T(y) = 1$, and the probability for joint passage is $\int_0^1 \int_0^1 S(x) \cdot T(y)\, dxdy$, which is equal to

$$(\int_0^1 S(x)dx)\cdot(\int_0^1 T(y)dy).$$

This, however, is just the product of the probability for particle (I) to pass $\tilde{S}$ multiplied by the probability for particle (II) to pass $\tilde{T}$. Hence this picture, which exactly corresponds to an indeterministic model, seems to lead straight to factorizability. Is there any room for non-independent correlations here, short of making the catalogue consulted by one particle a function of the barrier confronting the other one - an arrangement that would entail an exchange of information in violation of physical locality?

I think there is one possibility not exploited above and that would allow for correlations. It is to suppose that the source (i.e., $\lambda$) predetermines that whatever number one particle (randomly) selects, the other one does as well. So, the particular number chosen is not pre-determined, and hence the events of passage (or not) are undetermined. But <u>that the numbers chosen are identical</u> is pre-arranged at the source. If this were possible, then the joint probability would be

$$\int_0^1 S(x)T(x)dx$$

and, as in the $Q=3/10$, $Q'=1/10$ deterministic model of Section (II), (almost) any correlations would be possible for joint passage. In particular, factorizability would fail.

Indeed one can now see that the (apparent) inevitability of factorizability only comes from a necessary but unstated assumption; namely, that the event of one particle's randomly selecting a number is itself stochastically independent of the event of the other particle's randomly selecting a number. My hypothesis, that whatever the numbers selected they are the same, amounts to denying this implicit assumption of stochastic independence.

(Clearly if y = f(x), for any integrable function f, we can assign catalogues to produce the desired correlations. My assumption that f is the identity function merely simplifies the discussion.)

Borrowing from Leibnitz, let us say that two random devices are in harmony if their outcomes are regularly correlated. Then my suggestion is that an indeterminist model may treat the correlation experiments as showing the results of random devices whose harmony is established at the source of the particle-pairs. This suggestion requires neither particle to be aware of what is happening to the other; each behaves according to its own random selection and catalogue of response functions. Thus physical locality is respected, although factorizability fails.

The suggestion of random devices in harmony amounts to suggesting that there is a conservation law, established when the systems are together at the source, and which maintains some constant functional relation between the outcomes, regardless of the particular outcomes. It is, to be sure, an indeterministic conservation law. Nevertheless, it is like other conservation laws, in that it functions over large distances and without requiring the exchange of any causal signals. Of course, I cannot give a classical analog for random devices in harmony since, classically, indeterminism is false, and there are no random devices. For the same reasons, I cannot describe the mechanism whereby harmony is established and maintained. Rather, the idea of random devices in harmony should be taken as a paradigm (or "ideal of natural order") which can be used to give some structure to the pattern of pairs of events in an indeterministic

universe, in the same way that probabilities different from 1/2 are used to give structure to the patterns of single events. Thus, it seems to me that we can allow the indeterminist to "see" the behavior of anti-correlated spin-1/2 systems as an instance of random devices in harmony, and to use the machinery of that paradigm to help organise his indeterministic world. To rule out such devices because one cannot provide the mechanism of their operation or because, in some other respect, they stretch our (classically tutored) imagination, would seem an unduly limiting kind of <u>a priori</u>-ism. Should such devices-in-harmony become an integral part of a successful indeterminist theory, we should no doubt respond to the challenge of explaining how they work by pointing to well-known physical exemplars - like spin-1/2 systems - and responding, "They work like that!", coupling this with a systematic treatment of indeterministic conservation and invariance principles. It is only by ruling such a possibility out of court in advance that one can muster support for factorizability.

because they are factorizable

The most interesting fact about the possibility of random devices in harmony, however, is that even allowing their use the restrictions of physical locality still require the (non-factorizable) probabilities to satisfy inequality (CH), and hence enforce the canons of responsible indeterminism. For physical locality would require that the catalogue of response functions available to each particle depend only on that particle and its type $\lambda$, and not on the barrier to which its mate may have to respond. Moreover, corresponding to each possible barrier $\tilde{S}$ this catalogue must contain a response function $S(x)$ satisfying

$$\int_0^1 S(x)dx = p(S,\lambda)$$

Further still, corresponding to each possible barrier $\tilde{T}$ for its mate, the mate's catalogue must contain a response function T(x) such that

$$\int_0^1 T(x)dx = p(T,\lambda) \quad ,$$ <u>and</u> , in order to provide for the correct joint probabilities, we must have

$$\int_0^1 S(x)T(x)dx = p(ST,\lambda).$$

But, as one can see, these are exactly the requirements of responsible indeterminism; i.e., that the single and joint probabilities, for each $\lambda$, be realizable as the single and joint distributions of several bivalent random variables defined on a common space. It follows that the probabilities, for each $\lambda$, will be constrained by (CH), and hence that (CH) will limit the ensemble probabilities as well.

The upshot of this discussion is this. Physical locality is consistent with non-factorizability, but does require that one's indeterminism be responsible. And responsible indeterminism is only possible for single and joint probabilities (for crossing the various barriers) that satisfy (CH). Hence, in the special Q, Q' case, (SCH) remains necessary (and sufficient) for the joint probabilities of crossing the barriers, even in the absence of factorizability. Thus the move from determinism to indeterminism seems of no avail. Random devices in harmony, and all, there are still quantum correlation experiments whose correlations violate (CH).

## V. <u>Random Variables</u>

The framework of factorizable, determinist theories is nothing other than the common probabilistic framework of bivalent random variables. I have tried to show that even when liberated from the constraint of

factorizability, indeterministic theories are subject to the limitations of this same framework. Indeed the problem posed by the correlation experiments is only a particular case - one especially simple and amenable to experimental investigation - of a very general problem in quantum mechanics; the troublesome question of joint distributions and commuting observables. For Nelson ([10], Theorem 14.1) has proved the following general theorem: Iff observables $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n$ are not pairwise commuting, then if each observable $\tilde{A}_j$ is made to correspond to some random variable $A_j$ (where $A_1, A_2, \ldots, A_n$ are all defined on the same probability space) there is always some observable $\tilde{S} = \alpha_1\tilde{A}_1+\alpha_2\tilde{A}_2+\ldots+\alpha_n\tilde{A}_n$ formed as a linear combination of the $\tilde{A}_j$, with real coefficients $\alpha_j$, and some state $\psi$ such that the quantum mechanical average value of $\tilde{S}$ in $\psi$ (i.e., $(\psi,\tilde{S}\psi)$) is different from the probability space average value of the corresponding random variable $S = \alpha_1A_1+\ldots+\alpha_nA_n$.

To apply this result to the correlation experiments look at the quantum observables named by the arrangements

$\tilde{A}$, $\tilde{B}$, $\tilde{A}\tilde{B}$, $\tilde{A}\tilde{B}'$, $\tilde{A}'\tilde{B}$ and $\tilde{A}'\tilde{B}'$, and whose spectrum is $\{0,1\}$.

The response functions, and their products, are the corresponding random variables; i.e.,

$A'(\lambda)$, $B(\lambda)$, $A(\lambda)B(\lambda)$, $A(\lambda)B'(\lambda)$, $A'(\lambda)B(\lambda)$ and $A'(\lambda)B'(\lambda)$.

Notice that the quantum observables are not pairwise commuting, hence by Nelson's theorem there is some state and some linear combination whose average in that state differs from that of its random variable look-alike. In particular, Clauser and Horne look at the linear combination

$\tilde{S} = \tilde{A}\tilde{B} + \tilde{A}'\tilde{B} + \tilde{A}'\tilde{B}' - \tilde{A}\tilde{B}' - \tilde{A}' - \tilde{B}$ in the singlet state $\psi$ in which

all pairs are emitted. The inequality (CH) shows cases where

$$\langle\tilde{S}\rangle_{\psi} = Q(\tilde{A}\tilde{B}) + Q(\tilde{A}'\tilde{B}) + Q(\tilde{A}'\tilde{B}') - Q(\tilde{A}\tilde{B}') - Q(\tilde{A}') - Q(\tilde{B})$$

differs from $\langle S\rangle = P(AB) + P(A'B) + P(A'B') - P(AB') - P(A') - P(B)$.

?

Thus inequality (CH), and other Bell-type inequalities, provide particular instances of where the treatment of non-commuting observables as random variables will fail to duplicate the quantum mechanical statistics, as the various no-joint-distribution results - like Nelson's - would lead us to expect.

My purpose in calling attention to these more general results is in no way to downplay the importance of the Bell-related work. It is, rather, to try to identify those general features that interfere with attempts to capture the quantum statistics, using customary probabilistic models. It seems reasonably clear that no straightforward deployment of the framework of random variables will be adequate. For in that framework joint (and multiple) distributions will exist for all random variables. Distributions corresponding to non-commuting observables will inevitably constrain the statistics for the commuting observables (as in the derivation of (CH)) in ways not provided for quantum mechanically, where these joint distributions fail to exist. The expectation is that some difference from quantum mechanics will show up somewhere. Of course this line of argument is merely heuristic. But every attempt to use the apparatus of random variables, of which I am aware, has run into the expected difficulty. And that suggests the soundness of the heuristic, and it leaves only two options (unless despair at understanding quantum theory; i.e., instrumentalism,

counts as a third).

One option is to try using random variables in a restricted and, perhaps novel, way - some way that avoids allowing the shadow of non-commuting observables to fall on the commuting ones. The second option is not to treat the observables of quantum theory as random variables at all, and to devise instead a different probabilistic framework that does not commit one to joint distributions other than in exceptional cases. Difficult though it is, it seems to me that this second option holds out the most promise for our being able to understand the quantum statistics. In previous publications ([5], [6]) I have tried to elaborate on one idea of this sort. Here I want to offer an account of the correlation experiments along those lines. First, however, I shall exercise the first option and suggest how tinkering with the random variables framework may well permit one to get around the difficulty posed by (CH).

VI. Synchronization Models

If the events of passing (or not) the various barriers are represented as random variables (taking value 1 for pass and 0 for not), then (CH) constrains the possible probabilities for passage. We may, however, try to insert a wedge between a probability for passage and a probability for detecting the passage; clearly the latter may differ from the former if only because of detector inefficiency. Clauser and Horne [2] suggest a model for spin measurements that exploits this possibility, and that model works well enough to account for the data of all existing experiments and for any others where the detector efficiencies are sufficiently low.

But if we are convinced that the root of the problem has to do with the whole question of joint distributions, as I have suggested above, then focusing on detector inefficiencies may not seem a radical enough break with the random variables framework.

I want to suggest another possibility. It is that, quite generally, the quantum mechanical probability for finding simultaneous values for a pair $\tilde{S},\tilde{T}$ of commuting observables is simply not to be identified with the probability for those observables to take on those values. It is, rather, the probability for some "stronger" event E(ST), where the occurrence of E(ST) implies that $\tilde{S}$ and $\tilde{T}$ take on the values, <u>but not conversely</u>. Thus, if $\tilde{S}$ were represented by a random variable S, and $\tilde{T}$ by a random variable T, the quantum joint distribution for $\tilde{S}$ and $\tilde{T}$ would <u>not</u> be equal to the joint distribution of S and T; it would, rather, correspond to the distribution of some other random variable (ST) that corresponds to E(ST). In this way, we give up the requirement (JD) - which, you recall, is the condition of factorizability for a determinist theory. (See footnote 1.)

Thus we avoid using the joint distribution structure of the family of those random variables that correspond, pairwise, to commuting observables. The hope is that, by so doing, we avoid the shadow of the non-commuting observables and the consequences of the no-joint-distributions theorems.

One way to fill in this sketch for the correlation experiments would be this. (I would encourage the reader to come up with some other ways, so that we have several alternatives to think about.) Consider an experiment

where particle (I) encounters an $\tilde{S}$-barrier and particle (II) a $\tilde{T}$-barrier. Suppose we read the quantum probability for joint success, $Q(\tilde{S}\tilde{T})$, as follows:

> $Q(\tilde{S}\tilde{T})$ = the probability that particle (I) passes $\tilde{S}$ and particle (II) passes $\tilde{T}$, _and_ that they arrive at their respective detectors close enough in time to be recorded as arriving in coincidence.

Then the probability on the right hand side is the probability for an event whose occurrence implies that each particle passes its barrier, but not conversely. On this reading, $Q(\tilde{S}\tilde{T})$ involves more than the probability that (I) passes $\tilde{S}$ and (II) passes $\tilde{T}$. Hence to require that $Q(\tilde{S}\tilde{T})$ is just the ensemble average for such events of joint passage would be a mistake; i.e., it would be a mistake, on this reading, to require (JD). Clearly what must be done to retrieve $Q(\tilde{S}\tilde{T})$ is to introduce a new random variable, ST, interpreted as follows:

> $ST(\lambda) = 1$ iff in a pair of particles of type $\lambda$, particle (I) would pass barrier $\tilde{S}$ and particle (II) would pass barrier $\tilde{T}$, _and_ they would then arrive at their respective detectors close enough in time to be counted in coincidence.

It would then be reasonable to replace (JD) by the requirement that

(JP) $\qquad Q(\tilde{S}\tilde{T}) = \int_{\Lambda} ST(\lambda)\rho(\lambda)d\lambda.$

I shall refer to models of the correlation experiments that implement the preceding scheme as _synchronization models_. One way to construct such a model, explicitly, is as follows.

Start with a deterministic hidden variables model (without factorizability (JD), but retaining (RV)). Associate with each barrier $\tilde{Q}$ and type $\lambda$, for

which $Q(\lambda) = 1$, a symbol $t(Q,\lambda)$ that is either s, m, or f, and is interpreted so that

$t(Q,\lambda) = x$ iff a particle in a pair of type $\lambda$ that crosses barrier $\tilde{Q}$ would produce a count at its detector very slowly (x=s), very quickly (x=f), or in moderate time (x=m); respectively.

(I will abuse this interpretation and refer to the particles themselves as "slow", "fast" or "moderate". But the key idea is that $\lambda$ determines how quickly a particle can by-pass its barrier, if it does so at all, and then reach the detector.) Suppose, now, we make the following central assumption about what goes on in a correlation experiment containing such "retarded" or "advanced" particles; namely, we assume that *a "fast" particle at one detector is never counted as arriving in coincidence with a "slow" particle (of the same pair) at the other detector*.

In accord with this assumption for $\tilde{S} = \tilde{A}$ or $\tilde{A}'$ and $\tilde{T} = \tilde{B}$ or $\tilde{B}'$ define a random variable $ST(\lambda)$ with values either 0 or 1 as follows:

$ST(\lambda) = 0$ iff either $t(S,\lambda) = f$ and $t(T,\lambda) = s$, or $t(S,\lambda) = s$ and $t(T,\lambda) = f$, or $S(\lambda)\cdot T(\lambda) = 0$;

$ST(\lambda) = 1$, otherwise.

Then, on the given assumption, $ST(\lambda) = 1$ just in case both particles in a pair of type $\lambda$ would pass their respective barriers *and* be counted as arriving in coincidence at their detectors. So, averaging $ST(\lambda)$ over all $\lambda$, as in (JP), should produce the quantum corre~~c~~lations. If we now confine attention to the Q, Q' case where $Q(\tilde{A}) = Q(\tilde{B}) = Q(\tilde{A}') = Q(\tilde{B}') = 1/2$

and where $Q(\tilde{A}\tilde{B}) = Q(\tilde{A}'\tilde{B}) = Q(\tilde{A}'\tilde{B}') = Q$, and $Q(\tilde{A}\tilde{B}') = Q'$, then we can get around the (SCH) inequality, that is supposed to restrict the physically local models, and prove the following result.

Theorem S. There are synchronization models for all Q, Q' experiments, provided only that $0 \leqslant Q \leqslant \frac{1}{2}$ and $0 \leqslant Q' \leqslant 1/2$.

Proof. The synchronization model can be read from the table below.

$Q \leqslant Q'$

| $\lambda$ | $A(\lambda), t(A,\lambda)$ | $A'(\lambda), t(A',\lambda)$ | $B(\lambda), t(B,\lambda)$ | $B'(\lambda), t(B',\lambda)$ | $AB(\lambda)$ | $AB'(\lambda)$ | $A'B(\lambda)$ | $A'B'(\lambda)$ |
|---|---|---|---|---|---|---|---|---|
| $0 \leqslant \lambda \leqslant Q$ | 1,f | 1, f | 1, f | 1, f | 1 | 1 | 1 | 1 |
| $Q < \lambda \leqslant Q'$ | 1,f | 0 | 1, s | 1, f | 0 | 1 | 0 | 0 |
| $Q' < \lambda \leqslant \frac{1}{2}$ | 1,s | 1, s | 1, f | 1, f | 0 | 0 | 0 | 0 |
| $\frac{1}{2} < \lambda \leqslant \frac{1}{2} + (Q'-Q)$ | 0 | 1, f | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{1}{2} + (Q'-Q) < \lambda \leqslant 1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | $Q' \leqslant Q$ | | | | | | |
| $0 \leqslant \lambda \leqslant Q'$ | 1, f | 1, f | 1, f | 1, f | 1 | 1 | 1 | 1 |
| $Q' < \lambda \leqslant Q$ | 1, f | 1, m | 1, f | 1, s | 1 | 0 | 1 | 1 |
| $Q < \lambda \leqslant \frac{1}{2}$ | 1, s | 1, s | 1, f | 1, f | 0 | 0 | 0 | 0 |
| $\frac{1}{2} < \lambda \leqslant 1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Assuming that $\lambda$ is uniformly distributed on [0, 1] it is straightforward to check that, whether $Q \leqslant Q'$ or vice versa, (where all the integrals go from 0 to 1)

$\int A(\lambda)d\lambda = \int B(\lambda)d\lambda = \int A'(\lambda)d\lambda = \int B'(\lambda)d\lambda = \frac{1}{2}$ and that

$\int AB(\lambda)d\lambda = \int A'B(\lambda)d\lambda = \int A'B'(\lambda)d\lambda = Q$, whereas $\int AB'(\lambda)d\lambda = Q'$. Thus the quantum correlations are retrieved in the required manner; i.e., in satisfaction of (RV) and (JP).

Clearly such synchronization models implement a strategy that stays within the probabilistic framework of random variables, but avoids the Bell-type limitations - and the more general ones involved in the no-joint-distribution theorems - by giving up (JD), deterministic factorizability. In view of the general joint distribution results, as well as others along the same lines (see footnote 1),(JD) is precisely the requirement to be avoided. I do so here by suggesting a physical hypothesis, involving lag-times for each particle separately and, in effect, a minimum "coincidence count time" for the pairs. This circle of ideas requires no exchange of information about the barriers, nor any other instantaneous causal signals. For the minimum coincidence time is taken to be a constant of the detector-coincidence-counter arrangement; and the lag times are programmed at the source for each particle separately, and for all possible barriers for each particle.

Thus the synchronization models provide a deterministic way of accounting for the quantum correlations, a way that respects the requirement of physical locality, but where factorizability does not hold. Of course the physical hypotheses involved in these model may turn out to be amenable to experimental check (this is a question that I have not thought through), and they may even be disconfirmed in some experiments. Nevertheless, the

existence of these models already shows that we can have physical locality without factorizability, even in the deterministic case. And more, it also shows that any attempt to base factorizability on physical locality must involve physical assumptions over and above those contained in the assumption of physical locality itself.

Although the ideas surrounding synchronization can be extended to produce physically local indeterministic models for all cases, I shall not pursue that generalization here. Rather, let us look at the second option: abandoning the framework of random variables.

## VII. Prism Models

Since 1967,([4]) and a long time before I became aware of the locality problem as formulated by Bell ( [1] ), I have been pointing at the most salient feature of the probabilistic framework of quantum theory, and urging that work in the foundations of the theory respect it. It is, of course, that quantum theory does not treat its observables as random variables over a common space, for such treatment would entail the existence of joint distributions in all cases. Rather, quantum theory associates a probability measure with the range of possible values (the spectrum) of each observable, separately; different measures with different observables. The theory does not derive these measures from some one underlying distribution, in the manner of random variables. I call the pair consisting of an observable, together with a probability measure on its possible

values, a <u>statistical variable</u>. Quantum theory, then, treats its observables as statistical variables, not random variables, and specifies a general rule for when an n-tuple of such variables will have a well-defined multiple distribution, and what it will be. (See my [5] , Section 10 and [6] , Section VIII.)

The hidden variables program is based on neglect of this feature of quantum theory, and tries to assimilate it to something else. But while we can see, from a general point of view, why such a program gets off on the wrong foot; it is not so easy to set it straight. For the natural reaction would be to point out that, since quantum theory mandates different probability measures for different experimental arrangements ("observables"), we should not suppose the same distribution $\rho(\lambda)$ to apply to $\tilde{A}\tilde{B}$ <u>and</u> to $\tilde{A}\tilde{B}'$, <u>and</u> to $\tilde{A}'\tilde{B}$, <u>and</u> to $\tilde{A}'\tilde{B}'$ - for no two of these are compatible. Yet, short of the sort of communication between the particles at the barriers that would violate physical locality, it is difficult to see how the distribution of properties at the source could be altered to correspond to these distinct joint experiments.

In the remainder of this section I want to explore one possible response. It is a wild idea, yet probably not wild enough to do full justice to quantum theory. So, again, I urge the reader to think up more radical models of his own that will avoid the limitations of the random variables framework.

My idea is to introduce the concept of partial random variables (and

their joint distributions), and to try this setting for the correlation experiments. The general idea is straightforward. Consider a probability space $\Lambda$ with a density $\rho(\lambda)$. A partial random variable Q is a $\rho$-measurable, real-valued function whose domain $\sigma(Q)$ is some subset of $\Lambda$, where $\int_{\sigma(Q)}\rho(\lambda)d\lambda\neq 0$. We can then define the distribution $m_Q$ of Q as that conditional probability measure (on the Borel subsets S) satisfying

$$m_Q(S) = \text{Prob}\,[\,Q^{-1}\,(S)\ /\ \sigma(Q)\,]\quad ,$$

where Prob $(T) = \int_T \rho(\lambda)d\,\lambda$. The joint distribution $m_{Q,Q'}$ for a pair Q,Q' of partial random variables, where it exists, is the unique measure on the Borel subsets of $R^2$ satisfying

$$m_{Q,Q'}\ (S\text{x}T) = \text{Prob}\ [\ Q^{-1}(S)\cap Q'^{-1}(T)\,|\,\sigma(Q)\cap\sigma(Q')\,]\quad .$$

Notice that every partial random variable has a well-defined distribution. But a pair Q,Q' of partial random variables will have a joint distribution just in case

$$\int_{\sigma(Q)\cap\sigma(Q')}\rho(\lambda)d\,\lambda\neq 0.$$

The general idea, then, is to treat the observables of quantum theory as statistical variables by taking an observable to be a pair, consisting of a partial random variable Q together with its distribution $m_Q$.

All the machinery for so treating the correlation experiments is right at hand. Consider a deterministic model for such an experiment. The response functions were the random variables. To switch to the present framework we want these to be defined, not for every type $\lambda$, but only for some. Thus associate with each barrier $\tilde{S}$ a proper subset $\sigma(S)$,

of $\Lambda$, of non-zero probability, and a response function $S(\lambda)$ defined only on $\sigma(S)$, and taking 0 or 1 as values.

We can try to understand such a move along the following lines. The ordinary sort of deterministic model supposes that for every $\lambda$ the particles of that type are capable of responding to any barrier whatsoever. The response functions, then, show the expected responses. Here, however, we introduce the possibility that although every particle is capable of responding to some barriers, no particle is capable of responding to all of them. In saying that a particle is "not capable of responding" to a certain barrier, I do not mean simply that the particle will not cross the barrier. I mean something stronger and stranger. I mean that in an experiment that contains such a barrier the particle simply will not show up at all; it could neither be detected as passing nor as not-passing that particular barrier. It will be as though the encounter with the barrier destroys all trace of the particle having entered the experimental arrangement. Thus, in every experiment, what we detect will be all that can be detected, for that experiment, but not all that was there to begin with. This is the wild idea that motivates the move to partial random variables, and that will enable us to retrieve the quantum correlations from their joint distributions.

Although it might be appropriate to refer to models involving partial random variables as "partial models", a livelier terminology suggests itself along the following lines. From the present perspective we can identify what goes wrong in the random variables framework as the attempt to make do with exactly one common domain of definition for all the random

variables. Our idea here is to introduce several. Just as Occam's Razor cautions that it is vain to do with more what can be done with less, Karl Menger ([11]) calls the opposite maxim - that it is vain to try to do with fewer what requires more - a "prism principle". It appears that more than one common domain is required to account for the quantum correlations, so I shall refer to models that utilize more as prism models.

There are many, different ways of constructing prism models adequate for the correlation experiments. Here I shall be content to display just one such model for the Q, Q' case, and to discuss some of the issues associated with it.

Theorem P. There are prism models for all Q, Q' experiments, provided only that $0 \leqslant Q \leqslant \frac{1}{2}$ and $0 \leqslant Q' \leqslant \frac{1}{2}$.

Proof. Consider three step functions $f$, $g_Q$, $g_{Q'}$ defined on $[0, 1]$ as follows.

$$f(x) = \begin{cases} 1, & 0 \leqslant x \leqslant \frac{1}{2}, \\ 0, & \frac{1}{2} < x \leqslant 1; \end{cases}$$

and

$$g_\alpha(x) = \begin{cases} 1, & 0 \leqslant x \leqslant \alpha, \\ 0, & \alpha < x \leqslant \frac{1}{2}, \\ 1, & \frac{1}{2} < x \leqslant 1-\alpha, \\ 0, & 1-\alpha < x \leqslant 1; \end{cases}$$

for $\alpha = Q$ and for $\alpha = Q'$.

Notice that

$$\int f(x)dx = \int g_\alpha(x)dx = \tfrac{1}{2}, \text{ and}$$

$$\int f(x)g_\alpha(x)dx = \alpha \text{ ; for } \alpha = Q, Q' \text{ ; where the integrals go from 0 to 1.}$$

Let the types $\lambda$ range over the interval $0 \leq \lambda \leq 4$ and be distributed there uniformly; i.e., according to $\rho(\lambda) = \frac{1}{4}$. The (partial) response functions are then given by the following table (where value 1 means "would be detected" and value 0 means "would not")

| $\lambda$ | $A(\lambda)$ | $A'(\lambda)$ | $B(\lambda)$ | $B'(\lambda)$ |
|---|---|---|---|---|
| $0 \leq \lambda < 1$ | $f(\lambda)$ | UNDEFINED | $g_Q(\lambda)$ | $g_{Q'}(\lambda)$ |
| $1 \leq \lambda \leq 2$ | UNDEFINED | $f(\lambda-1)$ | $g_Q(\lambda-1)$ | $g_Q(\lambda-1)$ |
| $2 < \lambda \leq 3$ | $g_Q(\lambda-2)$ | $g_Q(\lambda-2)$ | $f(\lambda-2)$ | UNDEFINED |
| $3 < \lambda \leq 4$ | $g_{Q'}(\lambda-3)$ | $g_Q(\lambda-3)$ | UNDEFINED | $f(\lambda-3)$ |

Each response function $S(\lambda)$ is defined on a proper subset $\sigma(S)$ of the interval $[0, 4]$, consisting of three out of the four unit subintervals. Thus

$$\int_{\sigma(S)} \rho(\lambda) d\lambda = \tfrac{1}{4} \int_{\sigma(S)} d\lambda = 3/4,$$

for $S = A, A', B$ or $B'$. Then, in all these cases,

$$\text{Prob } [S(\lambda) = 1 | \lambda \varepsilon \sigma(S)] = \frac{\frac{1}{4} \int_{S^{-1}(1)} d\lambda}{\frac{1}{4} \int_{\sigma(S)} d\lambda} = \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}{3} = \tfrac{1}{2} .$$

So, all the single quantum probabilities for detection ($Q(\tilde{A})$, $Q(\tilde{A}')$, ...) are retrieved from the distributions of the response functions, considered as partial random variables. To obtain the joint probabilities notice that for $S=A$ or $A'$ and $T = B$ or $B'$, $\sigma(S) \wedge \sigma(T)$ is the union of two unit subintervals, one from $[0,2]$ and the other from $(2, 4]$ . Then one has that

$$\text{Prob}\ [S(\lambda)=T(\lambda)=1|\lambda\varepsilon\sigma(S)\cap\sigma(T)] = \frac{\frac{1}{4}\ 2\int_0^1 f(x)g_\alpha(x)dx}{\frac{1}{4}(\int_{\sigma(S)\cap\sigma(T)}d\lambda)} = \frac{2\alpha}{2} = \alpha;$$

where $\alpha=Q$ for $(S,T) = (A,B)$, $(A'B)$ or $(A',B')$, and

$\alpha=Q'$ for $(S,T) = (A,B')$.

Hence the quantum correlations are retrieved from the joint distributions of the response function, as required.

(This theorem is actually a special case of a more general result, to the effect that any family of "minimally consistent" single and multiple distribution functions can be represented as the single and multiple distributions of partial random variables on a common space.)

While it is clear from Theorem P that the quantum probabilities can be obtained via prism models, it may not be clear at all how the data obtained from a correlation experiment can be used to get the probabilities. For the correlation experiments turn out coincidence rates, and the probabilities must be obtained from these. The usual pattern of relationships, here, is this. One obtains the rate R of coincidence counts in an experiment in which no barrier is placed in the way of either particle, and the rate R(ST) when barrier $\tilde{S}$ is in position for particle (I) and barrier $\tilde{T}$ in position for particle (II). One then calculates the probabilities $Q(\tilde{S}\tilde{T})$ as

$$(7.1) \qquad Q(\tilde{S}\tilde{T}) = \frac{R(ST)}{R}.$$

Similarly, one obtains the coincidence rate R(S-) in an experiment with barrier $\tilde{S}$ in place for particle (I) and no barrier in place for particle (II),

and the coincidence rate R(-T) in an experiment with no barrier in place for particle (I) and barrier $\tilde{T}$ in place for particle (II). Then the single probabilities are calculated as

$$(7.2) \qquad Q(\tilde{S}) = \frac{R(S-)}{R} \qquad \text{and} \qquad Q(\tilde{T}) = \frac{R(-T)}{R}.$$

Can we accomodate these relationships to the data that one might expect if the underlying mode of production of the particle-pairs were that of a prism model? I do not think there is one, general answer to this question. For, the various prism models will differ from one another precisely in their implications for such coincidence rates. Moreover, there are different options one might take for how to treat experiments with no barriers. One could treat them as involving special "null" barriers, and enlarge the model by introducing new partial random variables to represent them. Or one could try to accomodate such experiments by means of general background assumptions.

Rather than trying to catalogue the general possibilities and responses (of which, anyway, I do not have a complete classification) I would prefer to explore one interesting option for the particular prism model just constructed. Indeed I chose this model just because the question of rates is especially difficult for it, and hence the response - I think - especially interesting.

In the model just constructed the rate R(ST) for any joint experiment with both barriers in place is ½ $Q(\tilde{S}\tilde{T})$; that is, only half of the emitted

particles show up at all in any such experiment. In order to maintain (7.1), then, the rate R (for a neither-barrier-in place experiment) must be $\frac{1}{2}$. But then, it follows from (7.2) that $R(S-) = \frac{1}{2}Q(\tilde{S})$, and $R(-T) = \frac{1}{2}Q(\tilde{T})$. Since we are only considering experiments where $Q(\tilde{S}) = Q(\tilde{T}) = \frac{1}{2}$, we must satisfy the following

(7.3) $R=\frac{1}{2}$; and $R(S-) = R(-T) = \frac{1}{4}$.

Here is my suggestion for how to do so. Suppose that every particle emitted by the source has a probability of 2/3 of being detected in an experiment where the particle encounters no barrier at all. (This is <u>not</u> to be understood as an hypothesis about ordinary detector inefficiency. What I postulate here is an intrinsic "quantum inefficiency". Thus those detectors that we should now call 100% efficient, could only detect 2/3 of the emitted particles - on this hypothesis.) Suppose, further, that the detection of particle (I), when that particle encounters no barrier, <u>is</u> the realization of a random selection x from [0, 1] where $0 \leqslant x \leqslant 2/3$. The detection of particle (II), when that particle encounters no barrier, <u>is</u> the realization of a random selection x from [0, 1] where either $0 \leqslant x \leqslant \frac{1}{2}$ or $2/3 < x \leqslant 5/6$. Then, in each case, the probability for detection would be 2/3, as required. But notice that if the particles were emitted in harmony (see Section IV) then the probability for their joint detection, when neither barrier is in place, would be 1/2. Hence, I shall assume that the particles are emitted in harmony, as above.

On this assumption the expected coincidence rate R, when neither barrier

is in place, would be $R=\frac{1}{2}$. In an experiment with $\tilde{S}$ in place for particle (I), we expect to count $3/4\ Q(\tilde{S}) = 3/8$ of the emitted particles. If there were no barrier in place for particle (II), then the probability for joint detection must be $(3/8)\cdot(2/3)=1/4$. (Notice that here no hypothesis of harmony makes sense, since the fate of particle (I) is predetermined at the source, in every case). Hence $R(S-) = \frac{1}{4}$. Similarly we have that $R(-T) = \frac{1}{4}$. Thus the rate requirements (7.3) are met, under these assumptions.

Here, then, is a detailed account of how the correlation experiments might run, realizing precisely the quantum probabilities as the ratios of coincidence rates in a manner consistent with the specifications of an underlying prism model. Before closing this discussion I want to point out one feature of this particular set of assumptions.

I have supposed that each particle has an intrinsic probability of only 2/3 for being detected, if it encounters no barrier. But, say, particle (I) of type $\lambda=\frac{1}{4}$ is certain to be detected if it encounters an $\tilde{A}$-barrier. (See the table of this section, under $A(\lambda)$.) Hence, according to the above assumptions, inserting a barrier between a particle and its detector will (in some cases) increase the probability of its being detected (from 2/3 to 1). Clauser and Horne ([2]) refer to such a possibility as "enhancement", and must reject it in order to carry out their derivations. (See also the discussion by Clauser and Shimony in[3], Section 5.4.1.) But it seems to me not an assumption that can be dismissed out of hand. Indeed, it seems especially tenable in the case of the

photon experiments, where the barrier is a polarizing plate, and the "particles" are bosons. For nothing there, so far as I know, warrants the assumption that the particle impinging on the source-side of the plate is the very same particle emitted from the detector-side. And if we think of them as different then, indeed, the emitted particle may be programmed for detection whereas the impinging particle may not have been so designed. I believe that the physics of polarization allows for such a possibility. Even in the case of fermions, if detectability is construed as an intrinsic, quantum property (as above) it seems possible to suppose that the interaction with a barrier (say a magnetic field gradient, in the case of charged particles) could enhance it in certain cases. What is at issue here, of course, is the microstructure of certain conservation laws (such as baryon charge conservation). The path of conventional wisdom directs us to re-impose that structure all the way down to the sub-quantum level. No one can say, however, whether this path leads to the truth of the matter. And, whatever that may be, it is one thing to claim that local causality rules out certain attempts to account for the quantum correlations. It is quite another thing to see that the operative principles concern the possible truth of some conjectured conservation laws.

## VIII. Conclusion

The conditions referred to as "locality" are requirements of factorizability. In the determinist case (Section II) it requires that

the quantum correlations be obtained as the joint distributions of those random variables which give rise to the single probabilities. In the indeterminist case (Section III) it is the requirement of stochastic independence, for each type of emitted pair. Examining the indeterminist case, I point out that no sound argument shows that genuine physical locality entails such factorizability, and that the plausibility of this requirement seems only to be grounded in intuitions of a determinist sort that appear to be out of place in the indeterministic setting. Moreover, if we conceive of indeterministic events as realizing certain random devices then it becomes possible to introduce the idea of harmony between such devices, and so to see how correlations may be achieved within the framework of physical locality (Section IV). The conclusion is that factorizability is not necessary for a physically local, indeterminist theory. I argue, nevertheless, that such a theory must be responsible, and that this requirement constrains the possible theories just as factorizability would. The mechanism of this constraint is the underlying conception of the experimental outcomes as random variables over a common probability space. In Section V I point out that the various Bell-type results are special cases of known limitations inherent in this random variables framework. The remainder of this essay is devoted to exploring two generic ways of avoiding the limitations of the framework, and to the development of particular determinist models of each type, the synchronization models of Section VI and the prism models of Section VII.

Acknowledgement. Work on this project was supported, in part, by National Science Foundation Grant SES 79-25917. I owe thanks to many people whose responses and criticism have influenced the development of the themes of this essay (and probably to others whose devastating criticism I never received); groups at Indiana University, Stanford and Chicago Circle have been particularly helpful. Abner Shimony has been a powerful and useful critic, Paul Teller a ready, if skeptical, ear, and Dana Fine has been my mainstay as assistant and counselor. Thanks to you all! (You too, M.)

## Footnotes

1. I have examined (JD) in [6], where I show that it is equivalent to requiring that the random variable corresponding to the quantum mechanical product observable $(\widetilde{ST})$ is the product $S(\lambda) \cdot T(\lambda)$ of the random variables corresponding, respectively, to the observables $\tilde{S}$ and $\tilde{T}$. This "product rule", if imposed for sufficiently many commuting observables $\tilde{S}$ and $\tilde{T}$, is by itself inconsistent with quantum theory (provided the values of observables are always well-defined and contained in their spectra), as Fine and Teller show ([9], Proposition 5). Thus (JD) is clearly the most dubious requirement, in the deterministic case. One plausible way of avoiding the product rule is contained in the discussion of Theorem S, Section VI below.

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